Diploma Programme subject outline — Group 5: Mathematics			
School name	**Gymnazium a SOS Rokycany**	**School code**	**061768**
Name of the DP subject *(indicate language)*	**Mathematics: applications and interpretation** **Language: English**		
Level *(indicate with X)*	Higher [ ] Standard completed in two years [X] Standard completed in one year * [ ]		
Name of the teacher who completed this outline	Vaclav Hofman	**Date of IB training**	12th – 14th February 2021
Date when outline was completed	05/2021	**Name of workshop** *(indicate name of subject and workshop category)*	Mathematics: Applications and Interpretation (Cat. 1)

* All Diploma Programme courses are designed as two-year learning experiences. However, up to two standard level subjects, excluding languages ab initio and pilot subjects, can be completed in one year, according to conditions established in the *Handbook of procedures for the Diploma Programme*.

1. Course outline

– Use the following table to organize the topics to be taught in the course. If you need to include topics that cover other requirements you have to teach (for example, national syllabus), make sure that you do so in an integrated way, but also differentiate them using italics. Add as many rows as you need.

– This document should not be a day-by-day accounting of each unit. It is an outline showing how you will distribute the topics and the time to ensure that students are prepared to comply with the requirements of the subject.

– This outline should show how you will develop the teaching of the subject. It should reflect the individual nature of the course in your classroom and should not just be a “copy and paste” from the subject guide.

– If you will teach both higher and standard level, make sure that this is clearly identified in your outline.

	Topic/unit (as identified in the IB subject guide) *State the topics/units in the order you are planning to teach them.*	**Contents**	**Allocated time** One class is 45 minutes. In one week there are 4 SL classes in Year 1 and 3 SL classes in Year 2.	**Assessment instruments to be used**	**Resources** *List the main resources to be used, including information technology if applicable.*
Year 1	**Topic 1—Number and algebra**	**Number and algebra** 1.1 **Operations with numbers** in the form $a\times10^k$ where $1\leq a<10$ and k is an integer. 1.2 **Arithmetic sequences and series**. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences. APP: Examples include simple interest over a number of years. Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life. 1.3 **Geometric sequences and series**. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for the sums of geometric sequences. APP: Examples include the spread of disease, salary increase and decrease and population growth. 1.4 **Financial applications** of geometric sequences and series: compound interest; annual depreciation. 1.5 **Laws of exponents** with integer exponents. Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology. 1.6 **Approximation**: decimal places, significant figures. Upper and lower bounds of rounded numbers. Percentage errors. Estimation 1.7 **Amortization and annuities** using technology. **Use technology to solve**: Systems of linear equations in up to 3 variables; Polynomial equations	25 hours (33 classes)	In this subject, both internally and externally assessed components of assessment are applied. Written examinations form the basis of the assessment. Students also complete in-school assessment tasks. Forms: Formative (mainly in-class instructions and helping and summative assessment (mainly written tasks, worksheets, in-class tests after each chapter)	IB SL Math course books, Khan Academy Math, YouTube videos, online articles, SW GeoGebra, SW Graph, GDC (other resources listed below)

		Chapter 1 Core: Exponents and logarithms 1A Laws of exponents.; 1B Operations with numbers in the form a x 10^k, where 1 <a<10 and k is an integer.; 1C Logarithms. Chapter 2 Core: Sequences 2A Arithmetic sequences and series.; 2B Geometric sequences and series.; 2C Financial applications of geometric sequences and series.			
	Topic 2—Functions	**Functions** **2.1 Different forms of the equation** of a straight line. Gradient; intercepts. Lines with gradients m1 and m2; Parallel lines m1=m2.; Perpendicular lines m1×m2=−1. **2.2 Concept of a function,** domain, range and graph. Function notation, for example f(x), v(t), C(n). The concept of a function as a mathematical model. Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line y=x, and the notation f−1(x). **2.3 The graph of a function**; its equation y=f(x).; Creating a sketch from information given or a context, including transferring a graph from screen to paper.; Using technology to graph functions including their sums and differences. **2.4 Determine key features of graphs**.; Finding the points of intersection of two curves or lines using technology. **2.5 Modelling with the following functions**: Linear models. f(x)=mx+c.; Quadratic models. $f(x)=ax^2+bx+c$; a≠0. Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y -axis.; Exponential growth and decay models. Equation of a horizontal asymptote.; Direct/inverse variation: $f(x)=ax^n$, n∈Z The y-axis as a vertical asymptote when n<0.; Cubic models: $f(x)=ax^3+bx^2+cx+d$.; Sinusoidal models: f(x)=asin(bx)+d, f(x)=acos(bx)+d. **2.6 Modelling skills**: Use the modelling process described in the "mathematical modelling" section to create, fit and use the theoretical models in section SL2.5 and their graphs.; Develop and fit the model: Given a context recognize and choose an appropriate model and possible parameters.; Determine a reasonable domain for a model.; Find the	38 hours (51 classes)	In this subject, both internally and externally assessed components of assessment are applied. Written examinations form the basis of the assessment. Students also complete in-school assessment tasks. Forms: Formative (mainly in-class instructions and helping and summative assessment (mainly written tasks, worksheets, in-class tests after each chapter)	IB SL Math course books, Khan Academy Math, YouTube videos, online articles, SW GeoGebra, SW Graph, GDC (other resources listed below)

		parameters of a model. Test and reflect upon the model: Comment on the appropriateness and reasonableness of a model.; Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation.; Use the model: Reading, interpreting and making predictions based on the model. Chapter 3 Core: Functions. 3A Concept of a function. 3B Sketching graphs			
	Topic 3—Geometry and trigonometry	**Geometry and trigonometry** **3.1 The distance between two points** in three-dimensional space, and their midpoint.; **Volume and surface area** of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids.; The **size of an angle** between two intersecting lines or between a line and a plane. **3.2 Use of sine, cosine and tangent ratios** to find the sides and angles of right-angled triangles. The sine rule: a/sinA=b/sinB=c/sinC; The cosine rule: c2=a2+b2−2abcosC; cosC=a2+b2−c2/2ab; Area of a triangle as 1/2absinC. **3.3 Applications of** right and non-right-angled **trigonometry**, including Pythagoras' theorem.; Angles of elevation and depression.; Construction of labelled diagrams from written statements. **3.4 The circle**: length of an arc; area of a sector. **3.5 Equations of perpendicular bisectors**. 3.6 **Voronoi diagrams**: sites, vertices, edges, cells.; Addition of a site to an existing Voronoi diagram.; Nearest neighbour **interpolation**.; Applications of the "toxic waste dump" problem. Chapter 4 Core: Coordinate geometry 4A Equations of straight lines in two dimensions. 4B Three-dimensional coordinate geometry Chapter 5 Core: Geometry and trigonometry 5A Volumes and surface areas of three-dimensional solids 5B Rules of trigonometry 5C Applications of trigonometry	25 hours (33 classes)	In this subject, both internally and externally assessed components of assessment are applied. Written examinations form the basis of the assessment. Students also complete in-school assessment tasks. Forms: Formative (mainly in-class instructions and helping and summative assessment (mainly written tasks, worksheets, in-class tests after each chapter)	IB SL Math course books, Khan Academy Math, YouTube videos, online articles, SW GeoGebra, SW Graph, GDC (other resources listed below)

	Topic 4—Statistics and probability 1/2	**Statistics and probability** **4.1 Concepts of population**, sample, random sample, discrete and continuous data.; Reliability of data sources and bias in sampling.; Interpretation of outliers.; Sampling techniques and their effectiveness. **4.2 Presentation of data** (discrete and continuous): frequency distributions (tables).; Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR).; Production and understanding of box and whisker diagrams. **4.3 Measures of central tendency** (mean, median and mode).; Estimation of mean from grouped data.; Modal class. Measures of dispersion (interquartile range, standard deviation and variance). Effect of constant changes on the original data.; Quartiles of discrete data. **4.4 Linear correlation** of bivariate data.; Pearson's product-moment correlation coefficient, r.; Scatter diagrams; lines of best fit, by eye, passing through the mean point.; Equation of the regression line of y on x.; Use of the equation of the regression line for prediction purposes.; Interpret the meaning of the parameters, a and b, in a linear regression y=ax+b. **4.5 Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event**.; The probability of an event A is P(A)=n(A)/n(U).; The complementary events A and A' (not A).; Expected number of occurrences. **4.6** Use of Venn **diagrams**, tree diagrams, sample space diagrams and **tables of outcomes** to calculate probabilities.; **Combined events**: $P(A \cup B)=P(A)+P(B)-P(A \cap B)$.; Mutually exclusive events: $P(A \cap B)=0$.; Conditional probability: $P(A \mid B)=P(A \cap B)/P(B)$.; Independent events: $P(A \cap B)=P(A)P(B)$. Chapter 6 Core: Statistics 6A Sampling 6B Summarizing data 6C Presenting data 6D Correlation and regression Chapter 7 Core: Probability 7A Introduction to probability. 7B Probability techniques	26 hours (34 classes)	In this subject, both internally and externally assessed components of assessment are applied. Written examinations form the basis of the assessment. Students also complete in-school assessment tasks. Forms: Formative (mainly in-class instructions and helping and summative assessment (mainly written tasks, worksheets, in-class tests after each chapter)	

Year 2	**Topic 4—Statistics and probability 2/2**	**Statistics and probability** **4.7 Concept of discrete random variables and their probability distributions**.; Expected value (mean), E(X) for discrete data. **4.8 Binomial distribution**.; Mean and variance of the binomial distribution. **4.9 The normal distribution and curve**.; Properties of the normal distribution.; Diagrammatic representation.; Normal probability calculations.; Inverse normal calculations **4.10 Spearman's rank correlation coefficient**, r_s.; Awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each.; **4.11 Formulation of null and alternative hypotheses**, H0and H1.; Significance levels. p -values.; Expected and observed frequencies. The $\chi 2$ test for independence: contingency tables, degrees of freedom, critical value. The $\chi 2$ goodness of fit test. The t -test. Use of the p -value to compare the means of two populations. Using one-tailed and two-tailed tests. Chapter 8 Core: Probability distributions 8A Discrete random variables. 8B Binomial distribution. 8C The normal distribution	19 hours (25 classes)	In this subject, both internally and externally assessed components of assessment are applied. Written examinations form the basis of the assessment. Students also complete in-school assessment tasks. Forms: Formative (mainly in-class instructions and helping and summative assessment (mainly written tasks, worksheets, in-class tests after each chapter)	IB SL Math course books, Khan Academy Math, YouTube videos, online articles, SW GeoGebra, SW Graph, GDC (other resources listed below)
	Topic 5—Calculus	**Calculus** **5.1 Introduction to the concept of a limit.; Derivative** interpreted as gradient function and as rate of change. **5.2 Increasing and decreasing functions**.; Graphical interpretation of $f'(x)>0$, $f'(x)=0$, $f'(x)<0$. **5.3 Derivative** of $f(x)=ax^n$ is $f'(x)=anx^{n-1}$, $n \in Z$ The derivative of functions of the form $f(x)=ax^n+bx^{n-1}+...$ where all exponents are integers. **5.4 Tangents and normals** at a given point, and their equations. **5.5 Introduction to integration as anti-differentiation of functions** of the form $f(x)=axn+bxn-1+....,f(x)=$	20 hours (27 classes)	In this subject, both internally and externally assessed components of assessment are applied. Written examinations form the basis of the assessment. Students also complete in-school assessment tasks.	

		$ax^n+bx^{n-1}+....$, where $n\in Z$, $n\neq -1n\in\mathbb{Z}$, $n\neq -1$. Anti-differentiation with a boundary condition to determine the constant term. Definite integrals using technology. Area of a region enclosed by a curve y=f(x)y=f(x) and the xx-axis, where f(x)>0f(x)>0. **5.6 Values of xx where the gradient of a curve is zero.** Solution of f'(x)=0f'(x)=0. Local maximum and minimum points. **5.7 Optimisation problems in context.** **5.8 Approximating areas using the trapezoidal rule.** <u>Chapter 9</u> Core: Differentiation 9A Limits and derivatives 9B Graphical interpretation of derivatives 9C Finding an expression for the derivative. 9D Tangents and normals at a given point and their equations. <u>Chapter 10</u> Core: Integration. 10A Anti-differentiation 10B Definite integration and the area under a curve Core SL content: Review Exercise Additional applications and interpretation SL content		Forms: Formative (mainly in-class instructions and helping and summative assessment (mainly written tasks, worksheets, in-class tests after each chapter)	
	The "toolkit" and Mathematical exploration	Skills of problem solving, proof, modelling, technology through the year.	30 hours (40 classes)		

2. IB internal assessment requirement to be completed during the course

Briefly explain how and when you will work on it. Include the date when you will first introduce the internal assessment requirement to your students, the different stages and when the internal assessment requirement will be due.

General information

Internal assessment is an integral part of the course and is compulsory for both SL and HL students. It enables students to demonstrate the application of their skills and knowledge and to pursue their personal interests, without the time limitations and other constraints that are associated with written examinations. The internal assessment requirements at SL and at HL are the same.

Scheduling (Year 2)

By the end of September, students are introduced to IA principles. In the following 9 weeks (first half of December), students are shown the sample IAs. Then students brainstorm ideas that match their interests and that they can imagine working on in their own IAs. At the end of December, students submit one to three selected topics and in the second week of January, they finally choose their topic. After this deadline, they are not allowed to change their choice of topic. By the beginning of March, students will submit their first draft. By the end of April, they will complete their final draft of the IAs.

Assessment criteria

During the process of evaluation, Presentation (4 marks), Mathematical communication (4 marks), Personal engagement (3 marks), Reflection (3 Marks), Use of mathematics (6 marks), will be considered.

For more detail see the Mathematics: applications and interpretation guide.

3. Links to TOK

You are expected to explore links between the topics of your subject and TOK. As an example of how you would do this, choose one topic from your course outline that would allow your students to make links with TOK. Describe how you would plan the lesson.

Topic	Link with TOK (including description of lesson plan)
Topic 1—Number and algebra:	Lesson Plan: Solving equations with technology OBJECTIVE: Students will be able to solve equations by undoing the operations in the reverse order of how the expression would have been evaluated. STUDENT INSTRUCTIONS 1) To introduce the lesson, use the critical component to understand: vocabulary – explain this word: equation 2) Show the steps on how to solve an equation. 3) Give an example of how to solve an actual problem. Exploring 1) Ask students how they would evaluate the expression: x/8 - 1/2 if they knew the value of x. 2) Ask students what is the first step in solving the equation. 3) Ask students what is the second step in solving the equation. Have students Think-Pair-Share. 4) Solve the two-step equation, such as x/8 - 1/2 = -7/2. PROBLEM-SOLVING: The four-colour problem is about how to colour maps on a flat surface. To prove it, mathematicians broke I down into a finite number of cases to consider, but there were far too many for it to be humanly possible, so they used a computer to check them all. Links to TOK: No human has ever checked all the cases, but they have checked the computer program which was used. Is this a valid way of proving something? Can you use some tools to prove it? How can you simplify the solution to this problem?

4. Approaches to learning

Every IB course should contribute to the development of students' approaches to learning skills. As an example of how you would do this, choose one topic from your outline that would allow your students to specifically develop one or more of these skill categories (thinking, communication, social, self-management or research).

Topic	Contribution to the development of students' approaches to learning skills (including one or more skill category)
Topic 2 – Functions Problem Solving, Use of GDC, social media, interpret the solutions etc.	**Thinking skills** Use prioritization and order of precedence in problem-solving **Social skills** Help others to create success for effective social interaction during group work **Communication skills** Organize and interpret data using both analogue and digital tools **Self-management skills** Practise focus and concentration while solving multiple problems **Research skills** Use a variety of technologies and media platforms, including social media and online networks, to source information

5. International mindedness

Every IB course should contribute to the development of international-mindedness in students. As an example of how you would do this, choose one topic from your outline that would allow your students to analyse it from different cultural perspectives. Briefly explain the reason for your choice and what resources you will use to achieve this goal.

Topic	Contribution to the development of international mindedness (including resources you will use)
Topic 3—Geometry and trigonometry: Voronoi diagrams	The Eurovision Song Contest is an international musical competition in which each country awards points to other countries. How could a Voronoi diagram help to explain some of the scoring? Are there any situations in which being geographically close differs from being culturally close? Students try to establish a link between mutual history, controversies and scoring patterns.

6. Development of the IB learner profile

Through the course it is also expected that students will develop the attributes of the IB learner profile. As an example of how you would do this, choose one topic from your course outline and explain how the contents and related skills would pursue the development of any attribute(s) of the IB learner profile that you will identify.

Topic	Contribution to the development of the attribute(s) of the IB learner profile
Topic 5—Calculus	**Thinkers:** Students make a mind map to attempt to solve the problem. Furthermore, students will apply this problem-solving method to situations which may occur in their daily lives. **Communicators:** Students will communicate in groups in order to discuss the possible ways to solve the problems at hand. **Risk-takers:** By working in groups, students will take risks at trying their own ways of solving the problem at hand, therefore, dealing with the uncertainty of the success of their way.

7. Resources

Describe the resources that you and your students will have to support the subject. Indicate whether they are sufficient in terms of quality, quantity and variety. Briefly describe what plans are in place if changes are needed.

The school's classrooms are all equipped with computers, multimedia projectors, touch boards, speakers, and high-speed Wi-Fi. There is a computer lab and a well-equipped library with several multimedia and VR stations accessible to students. The school has purchased teacher resource materials for every subject including textbooks, subject guides and teaching methodology material.
There is also a virtual link to the library of Western Bohemia University in Pilsen which enables students and teachers to use a wide variety of resources, magazine articles, fiction and non-fiction literature, etc.

There are also printing and scanning stations available to students and teachers enabling them to work with and create various teaching and learning materials.
Overall, the amount and quality of available resources is sufficient to give effective support to the Mathematics course.

Students use the school's library for research utilizing a link to the library University of West Bohemia, and to create a bibliography for their Internal Assessment. Also, the library has space and resources for teamwork on homework.
SL Mathematics course books from the school library, esp. P. Fannon, V. Kadelburg, B. Wooley, S. Ward: Mathematics, applications and interpretation, SL. Hodder Education. School library has a large number of mathematics books, both in English and Czech.
GD calculator: Casio fx-CG50 or Texas Instruments TI-Nspire CX or Texas Instruments TI-84 Plus C or similar (to be discussed and unified with Ss and Ts)
Web pages: IBO – resources and groups discussions; YouTube educative content; www.brightstorm.com/math/; www.mathsisfun.com/; https://ibo.org/; https://www.khanacademy.org/math; https://www.symbolab.com/; https://www.geogebra.org/.geogebra.; www.padowan.dk etc.
SW: Preferably: Graph, Desmos, Geogebra; similar SW is possible (to be tested)